The International Journal of Fracture Mechanics, Vol. 4, Nr. 3, September 1968.
Wolters–Noordhoff Publishing – Groningen.
Printed in the Netherlands.

A SIMULATED CRACK EXPERIMENT ILLUSTRATING THE ENERGY BALANCE CRITERION

S.J. Burns* and B.R. Lawn**

* Division of Engineering, Brown University, Providence, Rhode Island 02912, U.S.A.
** Now at School of Physics, University of New South Wales, Kensington, N.S.W., Australia.

ABSTRACT

The Griffith energy balance fracture criterion, despite its central importance in the study of most fracture problems, has received very little experimental confirmation. One of the reasons for this is the inherent gross uncertainty in the value of the fracture surface energy γ in solids. This paper describes a simple experiment in which the rupture of a thin liquid film between two glass strips forced apart by a wedge is used to simulate the propagation of a crack. The resistance to rupture is provided by the surface tension of the liquid, which can be computed by observing the configuration of the bent glass arms and then compared with values measured by independent means. Such a comparison is made for a silicone oil, for which agreement in values of γ is obtained within the limits of experimental scatter, thereby permitting a direct confirmation of the energy balance equations.

INTRODUCTION

In 1920 Griffith[1] postulated his now famous energy balance criterion, which states that a crack in a brittle solid will extend if the rate of release of strain energy is sufficient to overcome the resistance provided by the creation of new surface. Making use of the stress analysis of Inglis[2] for elliptical cavities Griffith applied the energy criterion to derive a relationship between the strength of a solid and the size of inherent flaws. Then by introducing flaws of known size into the surface of glass specimens Griffith went on to verify the form of this relationship. From the proportionality constant Griffith evaluated the fracture surface energy for his glass and concluded that this compared favorably with a value extrapolated from measurements of surface tension made near the softening point.

An alternative criterion for crack extension which has since received consideration is one which considers the mechanism of rupture at the crack tip. This criterion predicts that a crack should extend when the stress intensity near the crack tip becomes sufficient to overcome the cohesive forces resisting fracture. For a tensile crack in a brittle solid an equivalent statement is that the local stresses ahead of the crack tip are sufficiently intense to rupture atomic bonds and thus to create new free surface.

The question has been raised as to which of the two above fracture criteria, the macroscopic energy balance criterion or the mechanistic local crack tip stress intensity criterion, is the more fundamental. Barenblatt[3] argues that for an equilibrium crack in a brittle material the two approaches are self–consistent, and Rice[4] and Willis[5] confirm their equivalence rigorosly. Implicit in the treatment of the above authors is the assumption that the local tensile stresses ahead

of the crack tip do work such that all strain energy released by an extending equilibrium crack is expended only in creating new surface: under such conditions either fracture criterion is both necessary and sufficient.

This result has importance because if it can be established that there exists a mechanism by which the stresses at a crack tip effectively do work against a fracture resistance term, the energy balance principle may be applied to determine the equilibrium conditions for crack extension. Thus a quantitative analysis of the conditions at the crack tip, which is generally relatively complex, can be avoided. However, although the *forms* of the equations obtained by applying the energy balance principle to various equilibrium crack systems have received further confirmation since the work of Griffith, some doubt has been expressed as to the significance of the fracture surface energy term γ expressed in these equations. For the reversible surface energy of solids is not known sufficiently accurately, either theoretically or experimentally, to permit a reliable calibration of the energy balance equations. Further, dissipative processes, such as plastic flow, cleavage step formation, etc., often contribute to the uncertainty of the measured fracture surface energy. This paper describes a simple simulated crack experiment in which dissipative processes can be controlled and the γ term measured can be directly compared with independently recorded values. The degree of accuracy that can be attained between values of γ thus measured enables this arrangement to provide a simple and useful illustration of the energy balance principle.

EXPERIMENTAL ARRANGEMENT

Under the experimental arrangement used by Griffith (uniaxial tension) the equilibrium crack is unstable. Since then the techniques adopted for measuring fracture surface energies have tended to employ stable (and, in principle, reversible) cracks, thus permitting equilibrium conditions to be more accurately specified. Obreimoff[6] performed the first such experiment with controlled stable cracks by peeling thin strips of mica from a substrate block. He used simple beam theory to describe the geometry of the bent strip. Benbow and Roesler[7] and Gilman[8] extended this idea to thin cleavage specimens of plastics and crystals respectively. Measurements of γ using these 'cantilever' techniques are now standard procedure.

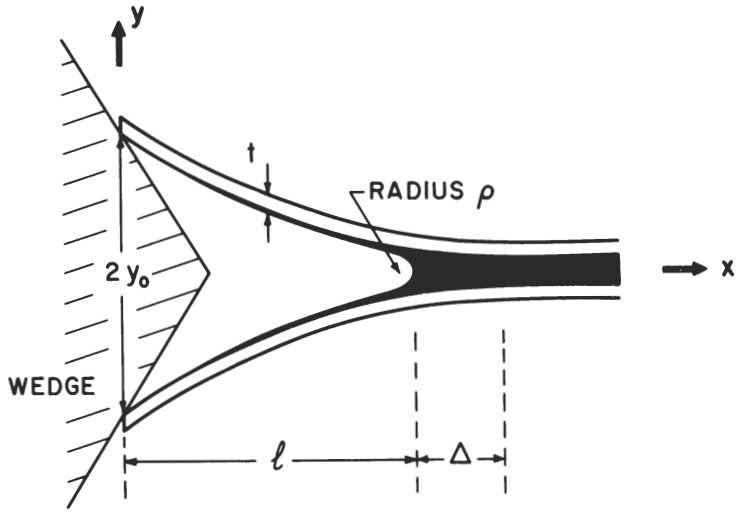

Fig. 1. Equilibrium configuration of glass arms wedged open at x = 0 and held together by a thin film of liquid beyond x = ℓ. $2y_0$ = mouth opening, t = thickness of beam, ρ = mean radius of curvature of liquid meniscus, Δ = distance beyond ℓ at which beams are effectively 'built–in'. The y scale is exaggerated.

The experimental arrangement used here is based on this cantilever principle. Two strips of glass 75 x 2.54 x 0.215 cm are thoroughly cleaned and then separated by a thin film of a wetting liquid (Fig. 1). The two arms are then forced apart by driving in a wedge, thus causing the liquid meniscus to move down the arms. The mechanism by which this simulated 'crack' advances may be understood by considering the line forces acting in direction BC of Fig. 2 on one half of the liquid

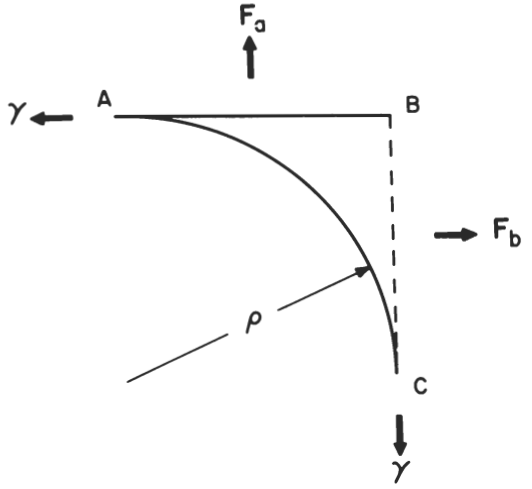

Fig. 2. Forces acting on half–section of near–cylindrical liquid meniscus. F_a is exerted at AB by the loaded arm, F_b is exerted at BC by the reduced pressure γ/ρ in the liquid, and the surface tension exerts forces γ at A and C.

meniscus. For equilibrium the line force F_a exerted by the loaded arm on the liquid at AB must, for zero contact angle at A, balance the surface tension γ at C. Then as the wedge is further inserted the increase in F_a gives rise to a driving force

$$F_1 = F_a - \gamma \tag{1}$$

which tends to move the liquid outward. Because of the molecular rearrangement which can occur within a liquid, rupture will not occur across any particular plane in the liquid (as it will in a solid) but will be manifested as a smooth depression of the meniscus as an increased number of molecules migrate to surface sites. Thus the force F_1 drives the 'crack' by working against the surface tension to create new surface of liquid until an equilibrium position is once again attained.

Having identified surface tension as the fracture resistance term we may now proceed to apply the energy balance principle to determine a relationship analogous to equation (1). Inserting the wedge between the glass arms introduces strain energy U_s into each half of the system, and this energy may then be reduced by allowing the crack to grow. Resisting this growth, however, is the work $U_\gamma = \gamma\ell$ required to create each new surface of liquid. The system is then in equilibrium when, for an incremental crack growth $\delta\ell$, the energy release δU_s just balances the energy gain δU_γ. For a non–equilibrium crack we may define a driving force F_2 per unit width of beam acting on each half of the liquid meniscus: we have

$$F_2 = \frac{\partial U_s}{\partial \ell} - \gamma \tag{2}$$

in analogy to equation (1).

If, now, we can calculate U_s as a function of ℓ the quantity γ may be computed from equation (2) when equilibrium conditions prevail. To do this we must first consider the geometry of the system. We take the arms to be simple beams wedged open at their mouth and held together beyond $x = \ell - \rho$ by the reduced pressure γ/ρ in the separating liquid due to the meniscus of mean radius ρ. The 'cantilever' arms are thereby taken to be effectively 'built–in' at some distance Δ beyond ℓ. Thus we impose the boundary conditions; $x = 0$, $y = \pm y_o$; $x = \ell - \rho$, $y = \pm\rho$; $x = \ell + \Delta$, $dy/dx = 0$. On the assumption that $\rho \ll y_o$, $\Delta \ll \ell$, we obtain from beam theory,

$$\frac{\Delta}{\ell} \approx \left(\frac{\rho}{y_o}\right)^{1/2} . \tag{3}$$

Typically $y_o \sim 0.5$ cm, $\rho \ll 10^{-4}$ cm, so that Δ/ℓ is generally less than one percent. Thus we can, to a good first approximation, consider the arms to be effectively built–in at $x = \ell$. Within this approximation the presence of the separating liquid is calculated to have negligible influence on the shape of the glass arms for $x < \ell$.* From simple beam theory the geometry of the arms is then described by the equation

$$\frac{2}{x}\left(1 - \frac{y}{y_o}\right) = \frac{3}{\ell} - \frac{x^2}{\ell^3} . \tag{4}$$

This equation has been investigated by observations of the configuration of the glass arms for two equilibrium crack lengths, with water as separating liquid. These observations are shown graphically in Fig. 3 as a linearized plot of $2(1-y/y_o)/x$ against x^2. The full lines in Fig. 3 are ob-

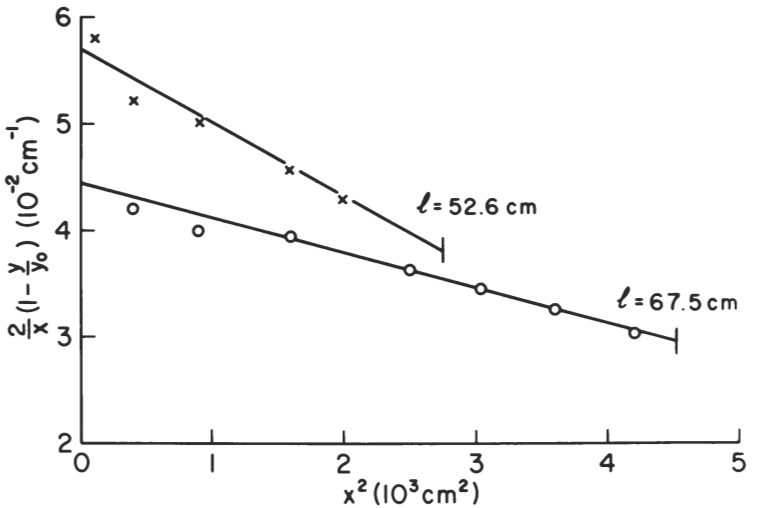

Fig. 3. Plot of $2(1-y/y_o)/x$ as function of x^2 for two equilibrium crack lengths, with water as the liquid film. The plotted points are derived from measured values of x, y and y_o, and the full lines are derived from measurements of y_o and ℓ only.

tained from measurements of y_o and ℓ only, while the points represent measurements of x and y/y_o along the glass arms. Coincidence of the two sets of data is obtained within the limits of experimental scatter.

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* Such effects may not always be negligible; for instance molecular forces of cohesion may have a significant effect on the shape of the crack tip contour in brittle solids.[3]

Having established the significance of the crack length ℓ we can now compute U_s from beam theory. We find

$$U_s = \frac{Et^3 y_o^2}{8\ell^3} \tag{5}$$

with E = Young's modulus, t = beam thickness. Combining equations (2) and (5) we have, for equilibrium (i.e. $F_2 = 0$),

$$\gamma = \frac{3Et^3 y_o^2}{8\ell^4} . \tag{6}$$

RESULTS

Experiments were attempted using a number of separating liquids. In each case the glass arms were suspended vertically to minimize gravitational effects, and the arm displacements were measured with a cathetometer. For each equilibrium setting enough excess liquid was squeezed out from between the arms to ensure that ρ was sufficiently small: this could be checked by observing the interference fringe pattern due to the narrowly separated glass arms near the crack tip contour. And in case the glass arms were slightly bent readings were repeated with the arms reversed.

Several factors combined to restrict the number of liquids for which useful results could be obtained. For instance, for heavy oils viscosity prevented equilibrium conditions from being reached within periods of days. The effects of viscosity[9] may be inferred by referring to the driving force concept introduced in equations (1) and (2). Far from equilibrium the driving force is large, and viscous resistance is overcome quite rapidly, but nearer to equilibrium this force becomes increasingly smaller and the approach to the equilibrium length becomes correspondingly slower. For light oils delay periods of several hours between readings proved necessary, and for water periods of several minutes were required. This delay time between readings led, in certain cases, to an added experimental difficulty because of the tendency for the liquid to evaporate from between the glass arms. Although this could be reduced by maintaining the entire system in a near–saturated atmosphere of the specimen liquid, with volatile liquids such as alcohol the evaporation rate was so great as to make any estimate of an equilibrium crack length uncertain. Evaporation

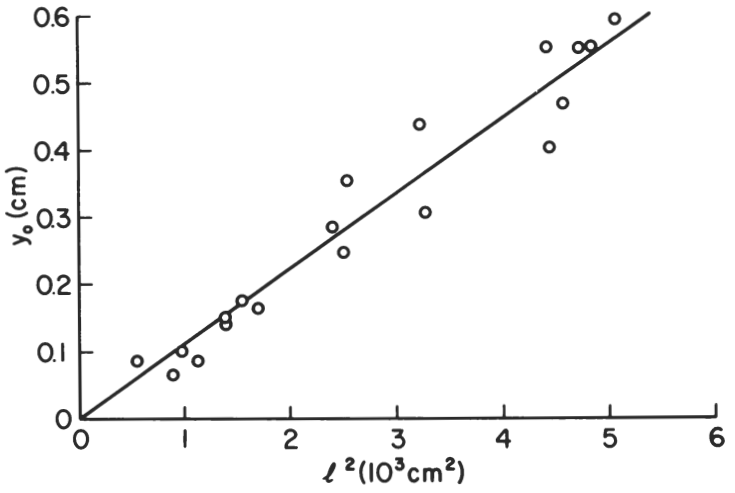

Fig. 4. Plot of y_o as function of ℓ^2 for a silicone oil. From the slope of this plot the surface tension of the liquid is obtained.

did not present a problem with liquids less volatile than water. Finally, liquids such as water proved to be extremely susceptible to contamination, thus precluding reproducible results. With these limitations in mind a silicone oil was chosen as the most suitable liquid for a concentrated study.

From equation (6) we see that the value of γ may be derived from the slope of a plot of y_o against ℓ^2. Such a plot is shown in Fig. 4 for the silicone oil. The points show a scatter somewhat greater than would be expected from the accuracy with which measurements could be recorded. The scatter is thought to arise from contamination of the liquid, but this has not been unequivocally established. The slope of the plot is, according to a least squares fit, $(1.12 \pm 0.05) \times 10^{-4}$ cm^{-1}. For the glass used $t = 0.215 \pm 0.002$ cm and $E = (7.2 \pm 0.1) \times 10^{11}$ dyn cm^{-2} (measured by observing the deflections of the beams under known loads) so that $\gamma = 34 \pm 4$ dyn cm^{-1}. This is to be compared with the surface tension of the liquid as measured by independent means. Here measurements are made by recording the heights of the liquid in capillary tubes, and also by recording the force required to withdraw a platinum ring from the liquid. A mean value of $\gamma = 31.3 \pm 1.4$ dyn cm^{-1} is obtained by these techniques. Thus agreement is obtained for γ within the limits of experimental scatter.

CONCLUSIONS

With the above simulated crack experiment agreement within a maximum uncertainty of less than 25% is obtained between those values of γ computed from the energy balance equation (6) and those measured by independent techniques. This compares favorably with the accuracy obtainable from the corresponding experiments on brittle solids, in which a factor of two between similarly compared γ values is considered acceptable. It was noted during the experiments performed here that the rate of crack growth became slower as equilibrium conditions were approached; for the silicone oil as the separating liquid a period of days was necessary for reasonable equilibrium conditions to be achieved, since non–reversible processes control the rate of crack growth.

It follows that the experimental arrangement outlined here may find value as a means for (a) demonstrating the validity of the energy balance principle, and (b) providing an illustrative model of crack propagation, in much the same way as the bubble raft model of Bragg and Nye[10] provides a pictorial analogue of dislocation movement in crystals.

ACKNOWLEDGEMENTS

The authors wish to thank Professors J.R. Rice and R.I. Tanner and Dr. B.L. Hunt for discussions. They also wish to acknowledge financial support from the Advanced Research Project Agency.

Received January 23, 1968.

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RÉSUMÉ – En dépit de son importance primordiale pour l'étude de la majorité des problèmes de rupture, le critère de rupture de Griffith basé sur un équilibre énergétique n'a été l'objet que d'un nombre limité de confirmations expérimentales.

Il faut en chercher l'une des raisons dans l'incertitude assez générale qui plane sur la valeur de l'énergie superficielle γ dans les solides. Le présent mémoire décrit un expérience simple, dans laquelle on fait usage d'un mince film liquide disposé entre deux plaques de verre que l'on sépare à l'aide d'un coin, pour simuler la propagation d'une fissure.

La résistance à la rupture est matérialisée par la tension superficielle du liquide, laquelle peut être calculée en observant la configuration prise par des plaques de verre fléchies, et en procedant par voie comparative.

Une telle comparaison a été effectuée dans le cas d'une huile a base de silicone. Dans les limites de la dispersion experimentale, on a obtenu des valeurs de γ plausibles, ce qui a permis d'établir une confirmation directe des équations d'équilibre énergétique.

ZUSAMMENFASSUNG – Das Griffith–kriterium für das Energiegleichgewicht beim Bruch hat bisher trotz seiner Bedeutung für die meisten Probleme in Bruchuntersuchungen kaum experimentelle Bestätigung gefunden. Eine der Ursachen dafür ist die äusserst grosse Unsicherheit bei der Bestimmung der Bruchflächenenergie γ von Festkörpern. In vorliegender Arbeit wird eine einfache Versuchanördnung beschrieben, in der die Bruchfortpflanzung mit Hilfe eines zwischen zwei schmalen Glasplatten liegenden Flüssigkeitsfilmes nachgeahmt wird. Der Bruch wird ausgelöst indem die beiden Glasplättchen durch einen Keil auseinander getrieben werden. Der Bruchwiderstand wird von der Oberflächenspannung der Flüssigkeit bestimmt, welche aus der Durchbiegung der Glasplättchen berechnet werden kann. Für Silikonol stimmen die derart gefundenen Werte für γ innerhalb der experimentellen Fehlergrenzen mit unabhängig davon gemessenen Oberflächenspannungswerten überein. Damit werden die für das Energiekriterium abgeleiteten Gleichungen direkt bestätigt.